1. INTRODUCTION

Flat slab structural systems have a large applicability due to their functional and economical advantages. These structures are known to be an elegant building system which is evidenced by the absence of beams and by its advantageous erection time in comparison with traditional frame buildings. Besides its obvious advantages, taken from engineering point of view the flat slab - column connection develops a complex behaviour. Close to ultimate states flat slabs are susceptible to punching. Under extensive loading stress distribution lead to a concentration of stresses near the column followed by a loss of strength across the connection. Existence of flexural and minute flaws influence, as well, the behaviour of the flat slab column connection zone. The influence is manifested by a diminished stress transfer capacity.

Punching of flat plates occurs without any warning and as a consequence of load boost showing extensive cracking and large deflections. One of the solutions in order to provide increase shear strength and higher rotational capacity at the flat slab - column connection is be made by introducing shear reinforcement in the control perimeter. In structural design of reinforced concrete flat slab – column connection the main idea is to ensure adequate rotational capacity to the connection zone, both for monotonic and cyclic loadings. This is made in order to avoid a non-ductile, shear, brittle failure. By providing fair amount of shear reinforcement a flexural failure is expected. Tough, flexural failures can trigger post-peak punching shear failures due to extensive cracking [2]. In order to avoid progressive collapse of these structures, punching failure must be ductile.

Figure 1.1 a) Standard flat slab – column connections b) different connection configurations c) Warehouse of Gerhard & Hey, St Petersburg; by Robert Maillart, 1912
Punching strength. Figure 1.2 presents a typical applied load – deflection for a flat – slab column connection. In case of brittle failure it can be observed that for ultimate force \( V_{u1} \), shear bearing capacity is lower than flexural capacity. In the other case, for connections with ductile failure it is vice-versa. In the second case the slab shows large deflections and rotations. Flexural failure is justified by the yielding of longitudinal reinforcement.

The ‘parents’ of flat slab column structures are: F. Hennebique (1894), C.A.P Turner (1905), A.L. Loleit (1907), swiss engineer Robert Maillart (1908) and german engineer 1912 who firstly proposed study commissions regarding this structural problem. The most important researchers that made experimental tests and provided analytical solutions to flat – slab column connections are: Talbot (1903), Kinnunen-Nylander (1960), Andrä (1990), Menetréy (1994), Hallgren (1996), Ozbolt et al (1999), Staller (2000) and Polak (1998).

\[ V_{flex} > V_{shear} \]

\[ V_{flex} \leq V_{shear} \]

**Figure 1.2 a) Load – deflection curves, b) Kinnunen & Nylander mechanical model, c) Andrä strut-tie model**

**Table 1.2 Primary objectives**

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<th>Experimental tests</th>
<th>Numerical modelling</th>
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<th>Secondary objective</th>
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<td>influence of:</td>
<td>- calibration of</td>
<td>- study of a database</td>
<td>Proposal of an analytical model for structural design, capable to account all the conclusion drawn after the experimental tests, numerical modeling and analytical studies</td>
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<td>- concrete strengths</td>
<td>experimental models</td>
<td>- effectiveness of shear reinforcement</td>
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<td>- transversal reinforcement</td>
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<td>- analysis of the tested specimens related to design codes</td>
<td>- influence of transversal reinforcement positioning and quantity</td>
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**2. SHEAR TRANSFER MECHANISMS**

\[ V_u = \rho \sqrt{f_y f_c \sin \theta} \left[ 1 + \sqrt{\frac{36.6}{d_e}} \left[ 1 + \frac{d}{19.4d_e} \right] \right]^{1/2} \]

**Figure 2.1 a) behaviour of cracked interfaces  b) size effect**
Concrete transfers shearing force between two cracked interfaces, in first phase, through the aggregate interlock mechanism. Mechanism compound by friction, adhesive bonding and interlock between the protruding aggregates. When another increment of loading is applied, the diagonal crack opens and the previous mechanism disappear rapidly. In the next phase, the shear in mainly transferred by the help of dowel effect of the reinforcement bars that cross the cracked interface. In case of this mechanism the shear is transferred though bond between the bars and concrete layer. Dowel effect is activated when the reinforcement bar crosses a concrete to concrete surface, that is subjected to slip. In the concrete layer surrounding the bars occurs a complex triaxial state of stresses. The dimension of the phenomenon depends on the reinforcement ratio, bar diameter, distance between bars, concrete cover and value of normal stresses in bars. Some of the researchers that studied to dowel effect are: Dulacska (1972), Pruïjssers (1988), Dei Poli (1992), X.G. He (2001), Ince (2007), El-Aris (2007).

Size effect is that phenomenon which shows that two elements of same geometry, designed and build at the same scale (dimensions, positioning of the reinforcement) will have different behaviour at failure if it is brittle. Relation proposed by Bažant (1984) for nominal strength, represents a gradual pass from an analysis al limit states (admissible stresses) to one based on fracture mechanics theory (figure 2.1c). Figure 2.3 presents the between the ξ parameter (size effect) and effective depth of the slab (d), representing different approaches given by several authors.

![Figure 2.2 Size effect approach by different authors](image)

### 3. SYNTHETIC PUNCHING MODEL

Starting from the information received from chapters 1 and 2, theoretical developments at the beginning of chapter 3, conclusions of experimental tests in chapter 4 and results of numerical modellings in chapter 5; the author proposes a bi-component synthetic formulation for assessment of punching capacity of flat slabs. The formulation is based on the concept of shear transfer mechanisms between concrete on cracked interfaces. It contains the share of concrete strut, dowel action, share of aggregate interlock and it is calibrated with the size effect. In contrast with design codes which consider that punching failure occurs when the stresses in concrete under diagonal direction is equal with concrete tensile strength, used as square root of $f_c$, the proposed model considers that longitudinal reinforcement, plays as well a very, if not, most important role in boosting the bearing capacity. Just few authors consider $f_y$ as a defining parameter in there models: Afhami (1997), Alexander and Simmonds (1992), Theodorakopoulos and Swamy (2002) [85,86,87,94].

**Concrete share**

$$V_{pun} = (V_{dow} + V_{ct}) \cdot \xi$$  \hspace{1cm} (3.11)

$$V_{ct} = \lambda_{sre} \cdot A_{sre} \cdot f_{yr}^{\frac{1}{3}}$$  \hspace{1cm} (3.12)

$$V_{dow} = \rho_i \cdot \sqrt{f_c} \cdot (f_y \cdot \sin(\alpha))^{\frac{1}{2}} \cdot d \cdot u$$ \hspace{1cm} (3.13a) - exact

$$V_{dow} = \rho_i \cdot \left( f_c \cdot f_y \cdot \sin(\alpha) \right)^{\frac{1}{3}} \cdot d \cdot u$$ \hspace{1cm} (3.13b) - design

Where

- $A_{sre} = \pi \cdot d \cdot (r_i + r_s)$ surface of punching cone
- $\theta$ – potential punching crack angle
- $f_c$ – uniaxial concrete compressive strength,
- $f_{yr}$ – uniaxial concrete tensile strength,
- $f_y$ – reinforcement yielding strength,
$r_1 = b_{cl}$, inferior diameter of punching cone

$w_2 = 2 \cdot \left( \frac{h}{2} + d \cdot \cot(\theta) \right)$, superior diameter of punching cone

$\lambda_{cpc} = d / r_2$, slenderness of the cone,

$\xi = \frac{1}{5} + \left( \frac{d}{l_{ch}} \right)^{-0.20}$, size effect for slabs

$l_{ch} = \frac{E_s G_F}{f_{yd}^2}$, characteristic cracking length

$G_F = 0.073 \cdot f_{cm}^{0.18}$ in mm/mm (fcm in MPa)

$0.004 < \rho_i < 0.02$, longitudinal reinforcement ratio

**Shear reinforcement share**

$$V_{sw} = \gamma_{sw} \cdot \sum A_{sw} \cdot \sigma_{sw} \cdot \sin \beta_{sw}$$ (3.14)

$$V_{pan} = (V_{dow} + V_{ct}) \cdot \xi + V_{sw}$$ (3.15)

$\sum A_{sw}$ - reprezintă aria de armătură care întâlneste conul potențial de străpungere dat de unghiul din relația (3.6)

$\beta_{sw}$ - unghiul armăturii transversale după perpendiculara la muchiile dalei

$$\sigma_{sw} = E_s \cdot e_{sw}, \quad \sigma_{sw} < f_{yw}$$ (3.16)

$$e_{sw} = l_{sw} / w_{cu,k} \cdot \cos(\theta_{crack})$$ (3.17)

**Maximum punching capacity**

$$V_{flex} = 2 \cdot \pi \cdot m_{k} \cdot \frac{r_s}{r_s - r_i} \cdot \frac{r_s}{r_s - r_i} = 0.20 \cdot \min(L_s, L_s)$$, flexural punching check

$$V_{strut} = \frac{f_{yd} \cdot A_{strut}}{\sin \theta_{strut}}$$, strut failure check

$$V_{max} = \min(V_{flex}, V_{strut})$$, maximum punching capacity

Figure 3.1 presents the approach of the model given to the punching phenomenon regarding: a) tangential stress distribution in slab considered on DB05 tested specimen, b) comparison with numerical results presented in chapter 5, c) comparison with punching test database from technical literature and d) comparison with design codes considering Model Code 1990, Model Code 2010, ACI-318:08, DIN-1045:2004 and Eurocode 2 (CEN 2004).
Six slab column connections have been tested during the experimental essay program. Four different configurations were essayed: for the same longitudinal reinforcement two shear reinforcement systems have been tested: double headed stud rails (DHSR) and stirrup beams (STwB). Table 4.1 presents geometrical characteristics and reinforcement system. Longitudinal reinforcement ratio was \( \rho_L = 0.5\% \). Average concrete cover was \( c_{\text{min}} = 15 \text{ mm} \), resulting in an effective depth of \( d = 155 \text{ mm} \). Column was reinforced with 8 bars Ø14 with Ø8/100mm stirrups. Transversal reinforcement was five perimeters of 12Ø10, \( A_{\text{stw}} = 9.425 \text{ cm}^2 \) each. Bearing of specimens was on rubber strips at a distance of 30 mm from the edges. Designed concrete strength was C20/25. Besides DB03 specimen, all other five elements showed higher compressive strengths that ones needed that concrete to be enframed in the early mentioned strength. Designed concrete strength had no influence on the results. Fact showed by comparing the ultimate results between DB02 and DB03 which are identical from all points of view, besides their concrete strength.

Since non of the shear reinforced slabs reached the punching predicted values by code design (generically written \( V_{\text{c}+V_{\text{y}}} \)), nor the predicted maximum punching capacity, an analysis regarding this issue was made. Maximum punching capacity is given by the relations on the right of figure 4.2. It can be observed that most of the codes put this value on concrete strength and on the crushing capacity of the concrete strut. It can be observed that the scatter of the results is quite big. EC2 and MC1990 show the most unsafe response, overestimating maximum punching capacity over 5 times. The justification comes from the strut and tie model and its angle of 45°. One the other hand MC2010 shows the best response, even in case of flexural failure, fact influenced the shape of the relation, which depends on the rotation of the slab. One can say that theory used by the EC2 and MC1990 is justified from the mechanical and theoretical point of view, but punching failure cannot predicted considering a development from the beam theory.

In case of the other two tested slabs, without shear reinforcement, it can be said, that the results were the ones expected and predicted by the designed codes. In both cases the ratios between the predicted values and the one recorded in tests are between 1.58 and 1.82, giving a safe response. Regarding the deflections and slab rotation at column faces, it has to be mentioned that the shear reinforced specimens showed a better behaviour with values up to 4 times higher than the ones without shear reinforcement. It has also been observed that in case of flexural failure, concrete strength had no influence on the results. Fact showed by comparing the ultimate results between DB02 and DB03 which are identical from all points of view, besides their concrete strength.
5. NUMERICAL ANALYSIS

Numerical tests have been divided in two main types: calibration models and sensitivity/parametric tests upon characteristic factors influencing flat slab – column connection behaviour. Constitutive models used were: concrete damaged plasticity for concrete and bi-linear elastoplastic for steel. In the first steps of analysis a calibration of the concrete constitutive model was made regarding dilation angle \( \psi \), eccentricity of the potential plastic surface \( m \), ratio between bi-axial and uniaxial concrete compressive strength \( f_c \), and the factor that influence the shape of the deviatoric stress plane \( \gamma \). The stress – strain curve for concrete in compression followed Park & Kent model. In analysis was accounted tension stiffening.

Numerical tests have been made as following: calibration of the model without shear reinforcement, influence of concrete strength on punching capacity for two longitudinal reinforcement ratios, influence of the effective depth of the slab for \( \rho_l = 1\% \), influence of longitudinal reinforcement ratio for same geometry, calibration of the models with shear reinforcement, influence and effectiveness of the shear reinforcement type, ratio and positioning, calibration of the model with opening in control perimeter.

![Figure 4.2 a) Maximum punching value b) Concrete punching capacity for DB05 & DB06](image)

6. CONCLUSIONS AND AUTHOR’S PERSONAL CONTRIBUTIONS

The following paragraphs summarize the results, conclusions and author’s personal contributions obtained during the doctorate research program, emphasizing the correlation between drawn conclusions and proposed objectives.

As a result of the experimental tests, numerical analysis and analytical studies few conclusions are drawn:
- punching failure occurs due to a combination of crushing the, compressed strut with rotation of the slab as a consequence of opening and propagation of the diagonal crack.
- flexural failure is mainly justified by the yielding of the bending reinforcement
- control specimen, DB05, without shear reinforcement, reached a value \( V_{u} = 495 \) kN with first cracks observed at 18.2% from the ultimate value, failure occurred at a shear strain of 1.75.\%
- DB06 specimen with opening in control perimeter showed an ultimate value representing 75% of the one recorded by the control specimen (i.e. 373.80 kN).
- providing shear reinforcement to the connection leads to higher ductility and a boost of strength; as a consequence, all specimens showed large deflections with tangential strains of 3.5% leading to a flexural failure.
- values can be organized regarding transversal reinforcement system: thereby DB02 și DB03 reached values 557.4 kN, respectively 561.3 kN (group ST-B); while DB01 and DB04, 517.20 și 527.40 kN (group DHSR).
- in the case of flexural punching and flexural failure, influence of concrete strength is very small compared to the one given by the reinforcement system.
- considering the effectiveness point of view, once the anchorage conditions are satisfied, stud – rails show a safer choice, both from ductility and technological point of view
- as a general conclusion it has to be said that thin slabs with d=155 mm made of normal strength concrete, designed strength C20/25, reinforced with small and medium longitudinal reinforcement ratios (e.g. $\rho=0.005$) show a failure mode defined by flexure
- as a conclusion of the previous studies, it has been observed that none of the design codes provided safe response regarding predicted values.
- as a consequence of the previous paragraph it must be said that EC2 shows an erroneous formulation regarding maximum punching capacity; adaptation of strut and tie model, derived from beam theory cannot be applied in the same shape for two-way shear.
- fib 2010 recommendations show the best response regarding this check
- bearing the laboratory specimens on rubber strips is unfavourable in processing results.
- even if the constitutive model for concrete is a complex and capable one, it is not fit with a failure criterion; in order to find the failure point, a subroutine has been written using Ottosen/MC2010 proposed failure criterion.
- as a consequence of calibration of DB05 specimen, whose behaviour was brittle, CDP constitutive parameters have been assessed as follows: dilation angle $\psi=32^\circ$, (for brittle behaviour values of $\psi$ have to be small ($\approx 20-25^\circ$), for ductile behaviour, it has to take large values $\approx 40^\circ$); eccentricity of the potential plastic surface, m, takes values around value of 0.5 being related to characteristic tensile strength and uniaxial and biaxial compressive strength; shape of the deviatoric stress plane takes value between 0.66$\rightarrow$1.00 (small values – high confinement, large values – low confinement).
- in order to find out the influence of $f_t$ at the maximum punching capacity, two sensitivity studies have been made; it has been found out that the punching shear capacity increases with concrete strength, failure resulted in a brittle way for all elements.
- loading rate influences behaviour and ultimate values of numerically modeled elements (i.e. loading rate – ratio between applied load $F$ and time in which it has been applied $t_i$), in order to find out the applied load a prediction using the design codes has to be made.
- in case of slabs with small and medium effective depth (ones used for buildings), $h_f < 300$ mm, development of the compressed strut take a linear shape, and failure occurs due to the development of high tangential strains
- in case of slabs with $h_f = 300$ mm – 450 mm, failure occurs due to the tension state of stresses that occurs in the punching cone, fact justified by the high effective depth and the possibility of appearance of a bigger compressed strut.
- slab with $h_f = 500$ mm, failed in flexure, due to the yielding strength of the reinforcement $f_t=400$ MPa, 
- angles of the compressed struts were as follows: for slabs with $h_f = 150$ mm - $\theta_{strut}=31^\circ$; for $h_f = 500$ mm - $\theta_{strut}=40^\circ$.
- applying the empirical relation proposed by the author regarding the assessment of the angle of the punching crack, the ratio between $\theta_{strut}/\theta_{crack}$ takes values between 1.00 for small values of effective depth and 1.15 for $h_f = 600$ mm
- for a yielding strength of $f_t=400$ MPa it has been observed that for longitudinal reinforcement ratios of $\rho=0.8\%$ and 1.4% response was the best (tangential strain was around $\varepsilon_{t,crack} = 2\varepsilon_{t,cc}^o$).
- in order to simulate correctly flexural failure behaviour, which is characterized by a strain hardening, second order isoparametric elements with 20 or 32 nodes have to be used; as well 8 brick 3D elements can be used, but hourglass control has to be applied
- in cease of DB01 and DB02 specimens, drawing the strut and tie sketch between concrete and transversal reinforcement, it has been observed that only the first reinforcement perimeter was in tension; result that consolidates the theory that only the perimeters crossing the potential failure plane have to accounted.
- stress concentration fields have developed following the slab diagonals (i.e. theoretical yield lines), thing showed by the fact that the higher stresses in transversal reinforcement was the one positioned under diagonal direction.
Personal contributions given by the author to the civil engineering and scientific society are: the proposal of the synthetic punching model (both in exact and design formulation). Formulation follows the mechanical principles of shear transfer mechanisms. Solutions have been given to:
- assessment of the potential punching angle
- application of relations for aggregate interlock mechanism
- exact formulation for assessment of the real punching strength, where concrete parameter influence is raised at $\frac{1}{2}$ power
- designed directed formulation, which offers complete verification relations regarding punching capacity
- assessment the share of the transversal reinforcement at ultimate capacity applying a fracture mechanics criterion and considering just the perimeters crossing the potential punching crack
- proposal of effectiveness factors for each transversal reinforcement system
- assessment of the maximum punching capacity for shear reinforced slabs as being the minimum between the value given by the failure of the compressed strut and the one given by the yield line theory $V_{\text{max}} = \min \left( V_{\text{flex}}, V_{\text{strut}} \right)$.

Other personal contribution regarding the experimental tests and numerical analysis:
- experimental research program and its conclusions
- testing of the specimens with transversal reinforcement system ST-B
- discussion regarding the lack of safeness provided by the code designs regarding maximum punching capacity check
- calibration of CDP model for flat-slab column behaviour
- clarifying problems regarding failure criterion + subroutine writing + work mode
- clarifying problems regarding behaviour of 3D solids (shear locking + hourglassing)
- results of numerical sensitivity tests.